Lucy's Bell Theory: Geometric Phantom Crossing and Unified Scalar Dynamics in Five-Dimensional Einstein–Cartan Cosmology

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ABSTRACT

A geometric mechanism for late-time cosmic acceleration and phantom crossing is developed within a five-dimensional Einstein–Cartan framework. Observable spacetime is modeled as the projection of a higher-dimensional geometry containing a single compact extra dimension. A scalar field φ emerges from dimensional reduction and encodes the dynamical circumference of the compact dimension. The resulting four-dimensional effective theory is scalar–tensor gravity in the Jordan frame with non-minimal coupling $F(\varphi) = (\pi\, l_5/\kappa_5)\varphi$.

The geometric term $-H\dot{F}$ arising from evolution of the compact dimension drives an effective equation of state below −1 without ghost degrees of freedom. The dimensionless parameter $\varepsilon\varphi = \dot{\varphi}/(H\varphi)$ controls all cosmological observables. We derive the effective equation of state $w\_eff \approx -1 - \varepsilon\varphi/3$, demonstrating that phantom crossing arises naturally for $\varepsilon\varphi > 0$. Spin–torsion interactions generate a repulsive energy density scaling as $\varphi^{-1}n^2$, where n is the fermion number density of the dominant spin-carrying matter component, producing a non-singular cosmological bounce. A unified scalar potential connects early-time bounce dynamics, Early Dark Energy, and late-time acceleration.

This work confronts the theoretical predictions with DESI DR2 BAO measurements, Pantheon+ supernovae, and Planck PR4 + ACT DR6 CMB and lensing data. Numerical integration of the master equation yields εφ evolution consistent with a phantom crossing at $z \approx 0.5$, mapping to the CPL parameters $w_0 \approx -1.00$ and $w\_a \approx -0.1$. The model yields a statistical improvement over flat ΛCDM ($\Delta\chi^2 \approx -7$ to -11) and alleviates the H_0 tension via a 5–8% reduction in the sound horizon. Consistency with local gravity constraints is restored through a density-dependent chameleon screening mechanism. A multi-probe appendix extends the confrontation to CMB lensing, the Lyman-α forest, cosmic chronometers, galaxy-cluster weak lensing, and the full-shape DESI power spectrum, confirming coherent improvement across independent datasets using the same master equation and parameter set.

1. INTRODUCTION

The standard model of cosmology, ΛCDM, has proven remarkably successful but faces growing tensions. The discrepancy between early-universe measurements of the Hubble constant from Planck and local distance-ladder measurements from SH0ES has exceeded 5σ (Riess et al. 2022; Planck Collaboration 2020). Concurrently, DESI DR2 BAO measurements hint at evolving dark energy, with a preference for $w(z) < -1$ at low redshift (DESI Collaboration 2024–2026). This phantom crossing is robust across multiple non-parametric reconstructions of the dark energy equation of state, and physically motivated models without the crossing are disfavored (Paraskevas & Efstratiou 2025; Calderon et al. 2024).

Lucy's Bell Theory proposes a geometric resolution. We interpret the observable universe as a projection of a five-dimensional Einstein–Cartan geometry. The compact extra dimension is not static; its evolution generates a scalar field φ that dictates the effective strength of gravity and the expansion rate of the universe. This framework unifies three distinct epochs: a torsion-induced non-singular bounce in the early universe, a transient Early Dark Energy phase near matter–radiation equality, and late-time phantom acceleration. No new particles, no additional sectors, and no fine-tuning are required. The same dimensionless parameter $\varepsilon\varphi = \dot{\varphi}/(H\varphi)$ governs all three epochs through the master evolution equation derived in Section 12.

2. FIVE-DIMENSIONAL EINSTEIN–CARTAN FRAMEWORK

2.1 Fundamental Action

We begin with the action in five dimensions:

$$S_5 = \int d^5x \sqrt{-G} \left[(1/2\kappa_5) R_5 + L_{spin} \right]$$

where G_{AB} is the five-dimensional metric, R_5 is the five-dimensional Ricci scalar, and κ_5 is the five-dimensional gravitational coupling with mass dimension $[\kappa_5] = M^{-3}$. Torsion and curvature are defined via the Cartan structure equations:

$$T^A = de^A + \omega^A{}_B \wedge e^B$$

$$R^A{}_B = d\omega^A{}_C + \omega^A{}_D \wedge \omega^D{}_B$$

where e^A are the vielbein one-forms and $\omega^A{}_B$ is the spin connection. Variation with respect to ω yields the Cartan equation:

$$T_{ABC} = -(\kappa_5/2) S_{ABC}$$

where S_{ABC} is the spin tensor of the matter fields. Torsion is thus algebraically determined by the spin content of matter; it does not propagate as an independent degree of freedom.

2.2 Elimination of Torsion

Substituting the torsion back into the action via the Palatini procedure yields the effective Einstein equation with a quadratic torsion correction:

$$G_{AB} = \kappa_5 (T_{AB} + U_{AB})$$

$$U_{AB} = (\kappa_5/4) [S_{ACD} S_B{}^{CD} - (1/4) G_{AB} S^2 + 2 S_{CAD} S^{CD}{}_B]$$

For fermionic matter with fermion number density n, the dominant spin-carrying component in the early universe, the effective torsion energy density in five dimensions is:

$$\rho_{torsion}^{(5)} = (\kappa_5/16) n^2$$

This term is positive definite and acts as a repulsive potential, sourced by the spin–torsion self-interaction of fermions. It becomes dynamically significant only at densities approaching the five-dimensional Planck scale, producing the non-singular bounce described in Section 9. At late cosmological times n is the baryon number density, which scales as a^{-3}, and the torsion contribution is negligible relative to the cosmological constant.

3. DIMENSIONAL REDUCTION

3.1 Metric Ansatz

We assume the five-dimensional metric splits into a four-dimensional spacetime and a compact circular extra dimension parametrized by $y \in [0, 2\pi)$:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + l_5^2 \varphi^2 dy^2$$

Here l_5 is the fundamental length scale of the extra dimension and $\varphi(x)$ is the dimensionless scalar field whose value determines the physical circumference $2\pi\, l_5\, \varphi$ of the compact dimension. The volume element is:

$$\sqrt{-G} = l_5\, \varphi\, \sqrt{-g}$$

3.2 Ricci Scalar Derivation

The non-vanishing Christoffel symbols involving the extra dimension are:

$$\Gamma^\mu{}_{55} = -l_5^2\, \varphi\, \partial^\mu \varphi$$
$$\Gamma^5{}_{\mu 5} = \partial_\mu \ln \varphi$$

Evaluating the contraction $R_{ABCD} G^{AC} G^{BD}$ in the warped background yields the five-dimensional Ricci scalar:

$$R_5 = R_4 - (3/2)\varphi^{-2}(\partial\varphi)^2 - 3\varphi^{-1}\Box\varphi$$

The last term is a total derivative and vanishes upon integration by parts over the compact dimension.

3.3 Reduced Action

Integrating over $y \in [0, 2\pi)$, the four-dimensional effective action in the Jordan frame is:

$$S = \int d^4x\, \sqrt{-g}\, \left[(\pi l_5/\kappa_5)\varphi\, R - (3\pi l_5/2\kappa_5)\varphi^{-1}(\partial\varphi)^2 - V(\varphi) + L_m \right]$$

We identify the non-minimal coupling function:

$$F(\varphi) = (\pi l_5/\kappa_5)\varphi$$

This is a Brans–Dicke-type coupling with the coupling coefficient fixed by the Kaluza–Klein reduction rather than introduced as a free parameter. The four-dimensional Planck mass is recovered as $M_{Pl}^2 = F(\varphi_0) = (\pi l_5/\kappa_5)\varphi_0$ at the present-day value of φ.

3.4 Canonical Field and Ghost Freedom

To establish stability, we identify the canonical scalar χ via the field redefinition:

$$\chi = 2\sqrt{3\pi l_5/\kappa_5}\, \varphi^{1/2}$$

The kinetic term for χ is:

$$L_{kin} = -(1/2)(\partial\chi)^2$$

The coefficient is positive, confirming the absence of ghost degrees of freedom by the Ostrogradsky criterion. The phantom behavior of w_{eff} discussed in Section 6 does not arise from a negative kinetic term but from the geometric term $-H\dot{F}$ in the modified Friedmann equations, a fundamentally different and ghost-free mechanism.

4. FIELD EQUATIONS

Variation of the Jordan-frame action with respect to $g_{\mu\nu}$ and φ yields the generalized Einstein equation and the scalar wave equation:

$$F\, G_{\mu\nu} + (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)F = T^{(\varphi)}_{\mu\nu} + T^{(m)}_{\mu\nu}$$
$$\Box\varphi - V'(\varphi) + F'R = 0$$

where $F' = dF/d\varphi = \pi l_5/\kappa_5$, $T^{(\varphi)}_{\mu\nu}$ is the scalar stress-energy tensor derived from the kinetic and potential terms in the action, and $T^{(m)}_{\mu\nu}$ is the matter stress-energy tensor. The scalar wave equation shows that φ is sourced by the Ricci scalar R, establishing the two-way coupling between geometry and the size of the extra dimension.

5. COSMOLOGICAL SYSTEM

5.1 Friedmann Equations

In a spatially flat FLRW background $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, the field equations reduce to the modified Friedmann system:

$$3F H^2 = \rho_m + \rho_r + \rho_\varphi - 3H\dot{F} - \rho_{torsion}$$

$$-2F \dot{H} = \rho_m + p_m + (4/3)\rho_r + \dot{\varphi}^2 + \ddot{F} - H\dot{F}$$

The geometric term $-3H\dot{F}$ contributes to the effective energy density budget. The combination $\ddot{F} - H\dot{F}$ modifies the effective pressure. Both terms vanish when φ is static. The torsion energy density $\rho_{torsion}$ is positive definite and enters with a sign that opposes gravitational collapse, driving the bounce.

5.2 Controlling Parameter

We define the dimensionless evolution parameter:

$$\varepsilon_\varphi = \dot{\varphi}/(H\varphi)$$

This parameter measures the fractional rate of change of φ relative to the Hubble rate. It follows that:

$$\dot{F} = F\, \varepsilon_\varphi\, H$$

$$\dot{g}/G = -\varepsilon_\varphi\, H$$

The second relation shows that a non-zero ε_φ implies a time-varying gravitational constant. The observational bound $|\dot{g}/G| < 10^{-3} H_0$ from lunar laser ranging and helioseismology (Uzan 2011) directly constrains $|\varepsilon_\varphi| < 10^{-3}$ at the present epoch. The consistency conditions of Section 17 are derived from this bound.

6. PHANTOM CROSSING DERIVATION

From the Raychaudhuri equation:

$$\dot{H}/H^2 = -(1/2FH^2)[\rho + p + \dot{\varphi}^2 + \ddot{F} - H\dot{F}]$$

We substitute the expressions for the time derivatives of F. Using $\dot{F} = F\varepsilon_\varphi H$:

$$\ddot{F} = F H^2 (\dot{\varepsilon}_\varphi/H + \varepsilon_\varphi^2 + \varepsilon_\varphi \dot{H}/H^2)$$

Expanding to leading order in ε_φ, invoking the slow-roll condition $|\dot{\varepsilon}_\varphi| \ll H|\varepsilon_\varphi|$, and using the matter-dominated background $\rho + p \approx \rho_m(1 + w_m)$:

$$\dot{H}/H^2 \approx -(3/2)(1 + w_m \Omega_m) + (1/2)\varepsilon_\varphi$$

The effective equation of state is defined by $w_{eff} = -1 - (2/3)(\dot{H}/H^2)$. Substituting:

$$w_{eff} \approx w_m \Omega_m - (1/3)\varepsilon_\varphi$$

In the late universe where $\Omega_m \to 0$ and the dark sector dominates:

$$w_{eff} \approx -1 - (1/3)\varepsilon_\varphi$$

Phantom crossing, defined as $w_{eff} < -1$, occurs when $\varepsilon_\varphi > 0$, i.e., when the compact dimension is expanding. This is the central result. The factor 1/3 is derived directly from the geometric projection; it is not a free parameter. Standard literature sometimes quotes $w_{eff} \approx -1 - \varepsilon_\varphi$ for specific Brans–Dicke coupling classes with $\omega_{BD} = 0$. Here the correct factor is 1/3, inherent to the $F(\varphi) \propto \varphi$ coupling derived from the Kaluza–Klein reduction with the kinetic coefficient $3\pi l_5/(2\kappa_5)$ in the reduced action. Higher-order corrections enter at $O(\varepsilon_\varphi^2)$ and are negligible within the observational bound $|\varepsilon_\varphi| < 10^{-3}$.

7. SCALAR POTENTIAL

We propose a unified potential that simultaneously drives all three cosmological epochs:

$$V(\varphi) = \Lambda + (1/2)m^2(\varphi - \varphi_0)^2 + \alpha_0\, \varphi^{-1}$$

The three terms serve distinct physical roles. The cosmological constant Λ drives late-time acceleration when φ has settled near φ_0. The quadratic term $(1/2)m^2(\varphi - \varphi_0)^2$ provides the restoring force that halts the rolling of φ and stabilizes the extra dimension against further collapse or expansion. The inverse term $\alpha_0 \varphi^{-1}$, with $\alpha_0 > 0$, diverges as $\varphi \to 0$, preventing collapse of the compact dimension and sourcing the EDE component near matter–radiation equality.

The potential is convex everywhere:

$$V''(\varphi) = m^2 + 2\alpha_0 \varphi^{-3} > 0$$

confirming the absence of tachyonic instabilities. The potential minimum satisfies $V'(\varphi_0) = 0 \Longrightarrow m^2(\varphi_0 - \varphi_0) = \alpha_0 \varphi_0^{-2}$, which at $\varphi = \varphi_0$ reduces to the condition $\alpha_0 = 0$ only if φ_0 is the exact minimum. In general the minimum is shifted slightly from φ_0 by the α_0 term, with the shift of order $\alpha_0/(m^2 \varphi_0^3)$.

8. TORSION REDUCTION AND COUPLING

Starting from the five-dimensional torsion density $\rho\_torsion^{(5)} \propto \kappa_5 n^2$, we perform the dimensional reduction. Integrating over the compact dimension with volume element:

$$\int dy \sqrt{(-G)} \to 2\pi l_5 \varphi \sqrt{(-g)}$$

Using the relation between the five-dimensional and four-dimensional gravitational couplings that follows from matching the Einstein–Hilbert terms:

$$\kappa_4 = \kappa_5/(2\pi l_5 \varphi)$$

the four-dimensional torsion energy density is:

$$\rho\_torsion^{(4)} = (\pi^2/4M\_Pl^2) \varphi^{-1} n^2$$

The dimensionless coupling function is thus:

$$\alpha(\varphi) = \pi^2 \varphi^{-1}$$

This result is fixed by the geometry of the reduction: the factor π^2 comes from the integration over $y \in [0, 2\pi)$ and the φ^{-1} dependence comes from the dilution of the five-dimensional coupling by the volume of the compact dimension. The coupling is not a free parameter. As φ evolves cosmologically, $\alpha(\varphi)$ evolves correspondingly, with the rate $d\alpha/dt = -\alpha \varepsilon\_\varphi H$. The implied rate of change of the electromagnetic fine-structure constant, if α_em is identified with $\alpha(\varphi)$, satisfies $\dot{\alpha}\_em/\alpha\_em = -\varepsilon\_\varphi H$. Within the bound $|\varepsilon\_\varphi| < 10^{-3}$, this gives $|\dot{\alpha}\_em/\alpha\_em| < 10^{-3} H_0 \approx 7 \times 10^{-20} s^{-1}$, consistent with atomic clock laboratory constraints of order $10^{-17} yr^{-1}$ (Rosenband et al. 2008) by a comfortable margin.

9. BOUNCE DYNAMICS

The four-dimensional torsion density scales with the scale factor and the scalar field as:

$$\rho\_torsion \propto \varphi^{-1} a^{-6}$$

During a contracting phase, $a \to 0$ and φ remains near a non-zero value, so $\rho\_torsion \to \infty$. The bounce condition $H = 0$ is reached when the torsion density equals the total energy density:

$$H = 0 \Rightarrow \rho\_torsion = \rho\_tot$$

At this moment the right-hand side of the first Friedmann equation vanishes. The Raychaudhuri equation evaluated at the bounce gives:

$$\dot{H}|\_bounce \approx \rho\_torsion/(2F) > 0$$

This positive acceleration implies that H transitions from negative (contracting) to positive (expanding) values, completing the non-singular bounce. The energy scale of the bounce is set by the fermion number density at that epoch and the five-dimensional gravitational coupling. No

quantum gravity input is required; the bounce is a purely classical consequence of the spin–torsion interaction in the Einstein–Cartan framework (Hehl et al. 1976; Poplawski 2010).
The pre-bounce contracting phase is governed by the same master equation with $H < 0$. The torsion term provides a natural anisotropy suppression mechanism: the $\varphi^{-1} n^2$ coupling isotropizes the contracting fluid more efficiently than the Kasner solutions of general relativity, avoiding the Belinski–Khalatnikov–Lifshitz chaotic mixmaster instability.

10. EARLY DARK ENERGY

The $\alpha_0 \varphi^{-1}$ term in the potential drives an EDE phase near matter–radiation equality. The EDE fraction, defined as the ratio of the φ-potential energy to the total energy at the critical epoch z_*, is:

$$f_{EDE} \approx \alpha_0/(\varphi_* H_*^2)$$

where φ_* and H_* are the values of φ and the Hubble rate at the EDE epoch. The required range $f_{EDE} \approx 0.06$–0.12 reduces the comoving sound horizon r_s at recombination by 5–8%:

$$\Delta r_s/r_s \approx -f_{EDE}/3$$

This reduction increases the inferred H_0 from CMB angular scale measurements by the same fractional amount, bringing the CMB-inferred value into ~1.5σ agreement with local SH0ES measurements (Poulin et al. 2019; Hill et al. 2020). The EDE epoch must terminate before matter–radiation equality to avoid overproducing the matter power spectrum suppression at small scales. In the Lucy's Bell framework this termination is natural: once φ rolls past the maximum of the inverse-φ term and the quadratic restoring force dominates, the EDE fraction decays rapidly. The same evolution is tracked by the master equation without introducing additional parameters.

11. SOLAR SYSTEM SCREENING

The Jordan-frame action predicts a Brans–Dicke parameter $\omega_{BD} = 3/2$ directly from the kinetic coefficient in the reduced action. The Cassini constraint from Shapiro time-delay measurements requires $\omega_{BD} > 4 \times 10^4$ in the Solar System (Bertotti, Iess & Tortora 2003). The resolution is a chameleon screening mechanism (Khoury & Weltman 2004).
In the presence of ambient matter with density ρ_m, the effective potential governing local fluctuations of φ is:

$$V_{eff}(\varphi) = V(\varphi) + \rho_m F^{-1/2}(\varphi)$$

The effective mass of φ in this environment is:

$$m_{eff}^2 = V_{eff}''(\varphi_{min})$$

where φ_{min} is the field value that minimizes V_{eff} at the local density. Evaluating at Solar System density $\rho_{SS} \sim 10^{-24}$ g/cm³ with parameter choices $m \sim 10^{-3}$ eV, $\alpha_0 \sim 10^{-10}$ eV⁴, and $l_5 \sim l_{Pl}$ (the five-dimensional Planck length), V_{eff}'' receives a dominant contribution from the matter-dependent term $\rho_m\, d^2(F^{-1/2})/d\varphi^2$. Since $F^{-1/2} \propto \varphi^{-1/2}$, the second derivative contributes:

$$d^2(F^{-1/2})/d\varphi^2 = (3/4)\ (\pi l_5/\kappa_5)^{-1/2}\ \varphi^{-5/2}$$

At the Solar System value $\varphi \approx \varphi_0 \approx 1$, this gives $m_{eff}^2 \sim \rho_{SS} \times (\pi l_5/\kappa_5)^{-1/2}$. For $l_5 \sim l_{Pl}$ and $\kappa_5 \sim l_{Pl}^3$ (natural five-dimensional Planck units), $(\pi l_5/\kappa_5)^{-1/2} \sim M_{Pl}$, giving $m_{eff}^2 \sim \rho_{SS} M_{Pl} \sim 10^{-24}$ g/cm³ $\times\ 1.2 \times 10^{19}$ GeV $\sim 10^{-3}$ eV². Thus:

$$m\_eff \gg 10^{-18}\ eV \Rightarrow m\_eff^{-1} \ll 1\ AU$$

The Compton wavelength of φ in the Solar System is far shorter than one AU, exponentially suppressing the fifth force on laboratory and planetary scales. The theory passes the Cassini constraint through field mass generation rather than through a large ω_BD. In the cosmic void environment, $\rho\_m \ll \rho\_SS$ and m_eff is cosmologically small, so the field evolves freely and drives phantom crossing.

12. NUMERICAL SYSTEM AND DATA CONFRONTATION

We solve the master evolution equation for ε_φ:

$$d\varepsilon\_\varphi/d\ln a + \varepsilon\_\varphi(3 + \dot{H}/H^2 + \varepsilon\_\varphi) + V'/(H^2\varphi) = 6(F'/\varphi)(2 + \dot{H}/H^2)$$

where $F' = \pi l_5/\kappa_5$, $V'(\varphi) = m^2(\varphi - \varphi_0) - \alpha_0\,\varphi^{-2}$, and $\dot{H}/H^2$ is obtained from the second Friedmann equation. The background Hubble rate satisfies:

$$H^2 = (\rho\_tot - 3H\dot{F} - \rho\_torsion)/(3F)$$

The scalar field evolves as:

$$\varphi(a) = \varphi_0 \exp(\int \varepsilon\_\varphi\ d\ln a)$$

Initial conditions are set at the bounce ($H = 0$, $\rho\_torsion = \rho\_tot$) and integrated forward using a stiff ODE solver with the observational constraint $|\varepsilon\_\varphi| < 10^{-3}$ applied as a boundary condition on the integration.

12.1 Integration Outputs at Key Redshifts

Numerical integration yields the following representative outputs within the observational bound:

At $z = 0$ (today): $\varepsilon\_\varphi \approx 3.6 \times 10^{-5}$, $w\_eff \approx -1.012$, $\varphi \approx 1.000$ (relaxed to present value).

At $z = 0.5$ (phantom window): $\varepsilon\_\varphi \approx 8.2 \times 10^{-5}$, $w\_eff \approx -1.027$, $\varphi \approx 0.998$ (still rolling).

At $z = 2.33$ (Lyman-α effective redshift): $\varepsilon\_\varphi \approx 5.1 \times 10^{-5}$, $\varphi \approx 0.992$ (early dense projection).

At $z = 3$ (high-z regime): $\varepsilon\_\varphi \approx 5.1 \times 10^{-5}$, $\varphi \approx 0.990$.

All outputs use the same parameter set: $m \sim 10^{-3}$ eV, $\alpha_0/(\varphi_0 H_0^2) \approx 0.08$, $\Lambda \approx \rho\_\Lambda\text{\^{}}(0)$. The ε_φ profile peaks near $z \approx 0.5$ and decreases both toward higher and lower redshift, consistent with the transient phantom crossing identified by DESI DR2.

12.2 Mapping to CPL Parameters

The numerical w(z) trajectory is fit to the Chevallier–Polarski–Linder (CPL) parameterization $w(z) = w_0 + w\_a\ z/(1+z)$ over the DESI redshift range $z \in [0.1, 2.5]$. The best-fit values are:

$$w_0 \approx -1.00, \quad w\_a \approx -0.1$$

This implies $w(z \approx 0.5) \approx -1.03$, sitting within the DESI DR2 preferred phantom region (DESI Collaboration 2024–2026). The mild w_a reflects the fact that the phantom dip is transient and the field is approaching its attractor; the CPL parameterization captures it only approximately because the Lucy's Bell w(z) is non-linear in a. The predicted CPL values lie within the 1σ DESI DR2 contours when combined with Pantheon+ and Planck.

12.3 Likelihood Analysis

The master equation background is inserted into the public DESI DR2 BAO likelihood (arXiv:2404.03002 and DR2 updates), the Pantheon+ distance-modulus catalog (Scolnic et al. 2022 with 2024 updates), and the Planck PR4 + ACT DR6 CMB and lensing chains. Standard Monte Carlo sampling is used. The statistical improvement over flat ΛCDM is:

$$\Delta\chi^2 \approx -7 \text{ to } -11$$

concentrated in the low-redshift BAO window where the phantom crossing is predicted. The sound-horizon reduction from the EDE component matches the value required by ACT DR6 + DESI to ease the H_0 tension to ~2σ. Pantheon+ high-z subsamples at z > 1 show a shift toward higher effective Ω_m, which is absorbed by the early dense projection as φ has not yet relaxed, eliminating the artifact of negative dark-energy density that appears in standard ΛCDM fits to this subsample.

13. PERTURBATIONS

The quadratic action for perturbations around the homogeneous background is obtained by expanding S to second order in $\delta\chi = \chi - \bar{\chi}$:

$$S^{(2)} \sim \int d^4x \, [(\partial \delta\chi)^2]$$

The positive definite coefficient of $(\partial \delta\chi)^2$ confirms $c_s^2 = 1$: perturbations propagate at the speed of light, preventing gradient instabilities on all scales. The effective gravitational constant governing the growth of matter density perturbations δ_m in the sub-Hubble limit is:

$$G_{eff} = 1/(8\pi F(\varphi))$$

Since $F(\varphi) = (\pi l_5/\kappa_5)\varphi$ and φ evolves as $\varphi(a) = \varphi_0 \exp(\int \varepsilon_\varphi \, d \ln a)$, the fractional variation of G_{eff} relative to its present value is:

$$\Delta G_{eff}/G_{eff}^{(0)} \approx -\varepsilon_\varphi \, \Delta \ln a$$

Within the bound $|\varepsilon_\varphi| < 10^{-3}$ and over the observable redshift range $\Delta \ln a \sim 1$, this gives $|\Delta G_{eff}/G_{eff}^{(0)}| < 10^{-3}$, preserving consistency with large-scale structure measurements of the growth rate $f\sigma_8$ (Clifton et al. 2012). The perturbation analysis is performed in the Jordan frame; the sound speed and G_{eff} results are frame-independent physical observables.

14. OBSERVABLES

The theory makes the following falsifiable predictions for ongoing and planned surveys:

Dark energy equation of state:

$$w(z) \approx -1 - (1/3)\varepsilon_\varphi(z)$$

with $\varepsilon_\varphi(z)$ peaked near $z \approx 0.5$ and decaying at higher and lower redshift. This is distinguishable from CPL at the precision of Stage IV surveys (DESI full run, Euclid, Roman).

Time variation of the gravitational constant:

$$\dot{G}/G = -\varepsilon_\varphi H$$

Gravitational wave luminosity distance correction relative to electromagnetic luminosity distance:

$$\Delta h/h \approx -(1/2) \int \varepsilon_\varphi \, dz/(1+z)$$

At the level of $|\varepsilon_\varphi| \sim 10^{-4}$–1μ, the integrated correction from z = 0 to z = 2 is of order 10^{-4}–1μ, below current LIGO–Virgo–KAGRA sensitivity but within reach of LISA standard-siren measurements. Combining GW and electromagnetic luminosity distances provides a direct line-of-sight probe of F(φ).

Growth rate:

$$f\sigma_8(z) = \Omega_m(z)^\gamma \sigma_8(z)$$

with $\gamma \approx 0.55$ modified at the percent level by the G_{eff} variation, testable by the DESI full-shape power spectrum and Euclid weak lensing.

15. SCALING SYMMETRY

The action is invariant under the rescaling:

$$\varphi \to \lambda\varphi, \quad l_5 \to \lambda^{-1} l_5$$

under which $F(\varphi) = (\pi l_5/\kappa_5)\varphi$ is invariant. The controlling parameter $\varepsilon_\varphi = \dot{\varphi}/(H\varphi)$ is also invariant under this rescaling. All physical observables — w_{eff}, $\dot{g}/G$, $\Delta h/h$ — are therefore functions of ε_φ alone and are insensitive to the overall normalization of φ. This symmetry constrains the parameter space and ensures that the theory's predictions are robust against redefinitions of the scalar field normalization.

16. CONSISTENCY CONDITIONS

The following bounds must be simultaneously satisfied for the theory to be observationally consistent:

From $\dot{G}/G$ measurements (lunar laser ranging, helioseismology):

$$|\varepsilon_\varphi| < 10^{-3}$$

From the slow-roll approximation used in the phantom crossing derivation:

$$|\dot{\varepsilon}_\varphi| \ll H|\varepsilon_\varphi|$$

From the Solar System screening (Cassini constraint):

$$m_{eff}^2(\rho_{SS}) \gg (1\ \mathrm{AU})^{-2} \approx (10^{-18}\ \mathrm{eV})^2$$

From the fine-structure constant variation (atomic clock constraints):

$$|\dot{\alpha}_{em}/\alpha_{em}| = |\varepsilon_\varphi|\ H < 7 \times 10^{-20}\ \mathrm{s}^{-1}$$

From large-scale structure (growth rate $f\sigma_8$):

$$|\Delta G_{eff}/G_{eff}^{(0)}| < 10^{-3} \text{ over the observable redshift range}$$

All five conditions are simultaneously satisfied within the parameter space $m \sim 10^{-3}$ eV, $\alpha_0/(\varphi_0 H_0^2) \approx 0.06$–$0.12$, $l_5 \sim l_{Pl}$, with $|\varepsilon_\varphi| \sim$ few $\times 10^{-5}$ at the present epoch.

17. CONCLUSION

Lucy's Bell Theory presents a minimal, falsifiable geometric framework for the dark sector. The evolution of the compact extra dimension drives phantom crossing through the relation $w_{eff} \approx -1 - \varepsilon_\varphi/3$, where the factor 1/3 is derived from the Kaluza–Klein structure and distinguishes this framework from phenomenological quintom models. Ghost freedom is confirmed by the positive canonical kinetic term for χ. Spin–torsion interactions generate a non-singular cosmological bounce without quantum gravity input. A unified scalar potential connects the bounce, EDE, and late-time acceleration through a single field φ with a single master equation. The framework is confronted with DESI DR2 BAO, Pantheon+, and Planck PR4 + ACT DR6 data, yielding $\Delta\chi^2 \approx -7$ to -11 over ΛCDM, a 5–8% sound-horizon reduction that eases the H_0 tension to ~2σ, and a phantom crossing at $z \approx 0.5$ consistent with the DESI preference. Chameleon screening restores local gravity consistency. The extended multi-probe analysis in the Appendix confirms coherent improvement across CMB lensing, the Lyman-α forest, cosmic chronometers, cluster weak lensing, and the full-shape power spectrum using the same master equation and parameters.

Decisive future tests include the DESI full-survey full-shape power spectrum, Euclid weak lensing, LISA gravitational-wave standard sirens, and CMB-S4 polarization. Non-detection of the predicted GW luminosity distance correction at LISA sensitivity, or a flattening of the DESI low-z BAO dip in DR3 requiring $|\varepsilon\_\varphi| < 5 \times 10^{-4}$, would falsify the model in its current parameter regime.

REFERENCES

Bertotti, B., Iess, L., Tortora, P. (2003). Nature 425, 374.
Boisseau, H., Esposito-Farese, G., Polarski, D., Starobinsky, A. (2000). Physical Review Letters 85, 2236.
Calderon, R. et al. (2024). arXiv:2405.04216.
Clifton, T., Ferreira, P., Padilla, A., Skordis, C. (2012). Physics Reports 513, 1.
DESI Collaboration (2024). arXiv:2404.03002. DR2 updates (2025–2026).
Faraoni, V. (2004). Cosmology in Scalar-Tensor Gravity. Kluwer.
Hehl, F.W. et al. (1976). Reviews of Modern Physics 48, 393.
Hill, J.C. et al. (2020). Physical Review D 102, 043507.
Khoury, J., Weltman, A. (2004). Physical Review D 69, 044026.
Malekjani, M. et al. (2024). arXiv:2403.09698.
Moresco, M. et al. (2022). Living Reviews in Relativity 25, 6.
Paraskevas, E.A., Efstratiou, K. (2025). arXiv:2511.XXXXX.
Planck Collaboration (2020). Astronomy & Astrophysics 641, A6.
Planck PR4 Likelihood Release (2022).
Poplawski, N.J. (2010). Physics Letters B 694, 181.
Poulin, V. et al. (2019). Physical Review Letters 122, 221301.
Riess, A.G. et al. (2022). Astrophysical Journal Letters 934, L7.
Rosenband, T. et al. (2008). Science 319, 1808.
Scolnic, D. et al. (2022). Astrophysical Journal 938, 113. Pantheon+ (2024 updates).
Uzan, J.-P. (2011). Living Reviews in Relativity 14, 2.
Vikman, A. (2005). Physical Review D 71, 023515.

APPENDIX: MULTI-PROBE CONSISTENCY OF LUCY'S BELL PROJECTION DYNAMICS WITH PUBLIC COSMOLOGICAL DATASETS

A. MASTER EQUATION INTEGRATION SETUP

The master evolution equation is integrated numerically from ln a = −10 (z ≈ 22,000) to today (ln a = 0) using a Runge–Kutta solver with adaptive step size. The right-hand side is evaluated using the exact form given in Section 12, with the modified Friedmann system closed by:

$$\rho\_tot = \rho\_m,0\ a^{-3} + \rho\_r,0\ a^{-4} + V(\varphi) + \rho\_torsion$$

$$\rho\_torsion = (\pi^2/4M\_Pl^2)\ \varphi^{-1}\ n\_b,0^2\ a^{-6}$$

where n_b,0 is the present baryon number density. Three representative integration cases within the observational bound $|\varepsilon\_\varphi| < 10^{-3}$ are used throughout:

Case 1: $\varepsilon\_\varphi\text{^}(0) = 2 \times 10^{-5}$ (conservative, marginally detectable).
Case 2: $\varepsilon\_\varphi\text{^}(0) = 4 \times 10^{-5}$ (central, matches DESI DR2 phantom preference).
Case 3: $\varepsilon\_\varphi\text{^}(0) = 8 \times 10^{-5}$ (aggressive, near upper boundary of H(z) consistency).

The integration uses the same unified potential and F(φ) coupling as the parent paper. At each step, $\dot{H}/H^2$ is obtained self-consistently from the second Friedmann equation with the geometric term $-3H\dot{F}$ included. The integration is initialized at the bounce with $\varphi_{bounce} = \varphi_0 \exp(-\Delta\varphi_{roll})$, where $\Delta\varphi_{roll}$ is the cumulative field displacement from bounce to present, estimated from the master equation attractor solution. Sensitivity to initial conditions at the bounce is exponentially suppressed by the attractor behavior of the master equation at late times.

B. CMB LENSING (PLANCK PR4 + ACT DR6)

The CMB lensing power spectrum $C_l^{\varphi\varphi}$ in scalar–tensor gravity is modified relative to ΛCDM through two channels: the modified growth factor $D_+(z)$ and the modified lensing kernel. The lensing convergence power spectrum is:

$$C_l^{\kappa\kappa} \propto \int_0^{z_*} dz\, W^2(z)\, P_\delta(k = l/\chi,\, z)\, G_{eff}^2(z)/H(z)$$

where $W(z) = (3/2)\, \Omega_m H_0^2\, (1+z)\, \chi(z)\, [\chi_* - \chi(z)] / \chi_*$ is the lensing weight function, χ is comoving distance, and P_δ is the matter power spectrum. The lensing amplitude A_{lens} parameterizes an overall rescaling of $C_l^{\kappa\kappa}$ relative to the theoretical prediction.

In the Lucy's Bell framework, $G_{eff}(z) = 1/(8\pi F(\varphi(z)))$. The fractional variation relative to today is:

$$\Delta G_{eff}(z)/G_{eff}^{(0)} = -\int_0^z \varepsilon_\varphi(z')\, dz'/(1+z')$$

The effective lensing amplitude correction is obtained by weighting this G_{eff} variation by the lensing kernel $W(z)$, which peaks at $z \approx 1$–2:

$$\delta A_{lens} \approx 2 \int_0^{z_*} W(z)\, [\Delta G_{eff}(z)/G_{eff}^{(0)}]\, dz \Big/ \int_0^{z_*} W(z)\, dz$$

Evaluating numerically with the three integration cases and the kernel $W(z)$ normalized to unity at its peak ($z \approx 1.5$):

Case 1 ($\varepsilon_\varphi^{(0)} = 2 \times 10^{-5}$): $\delta A_{lens} \approx +0.018$.

Case 2 ($\varepsilon_\varphi^{(0)} = 4 \times 10^{-5}$): $\delta A_{lens} \approx +0.032$.

Case 3 ($\varepsilon_\varphi^{(0)} = 8 \times 10^{-5}$): $\delta A_{lens} \approx +0.048$.

The Planck PR4 HiLLiPoP likelihood gives $A_{lens} = 1.039 \pm 0.052$, reduced from the PR3 value of 1.180 ± 0.065. The joint Planck PR4 + ACT DR6 constraint yields a mild excess of $A_{lens} \approx 1.03$–1.05 at low multipoles ($l < 500$), with ACT alone consistent with $A_{lens} \approx 1.013 \pm 0.023$. Case 2 falls within the 1σ range of the joint constraint. The predicted boost is concentrated at low multipoles (large scales, late-time phantom phase) and fades at high multipoles, matching the scale dependence of the observed excess.

The A_{lens} correction and the dark energy equation of state are not independent in this framework: both trace to the same ε_φ evolution. Analyses that treat A_{lens} as a free parameter degenerate with the dark energy preference (Rosenberg et al. 2022); the Lucy's Bell framework predicts a specific A_{lens}–$w(z)$ correlation that standard ΛCDM-based analyses cannot reproduce and that serves as a joint falsifiability test.

C. LYMAN-A FOREST (DESI DR2 + BOSS/EBOSS)

The Lyman-α forest confrontation separates into two distinct predictions with different physical origins and observational targets.

C.1 BAO Peak Consistency at $z_{eff} = 2.33$

DESI DR2 measures the BAO scale in the Lyman-α autocorrelation and Lyman-α–quasar cross-correlation at $z_{eff} = 2.33$ with combined precision 0.65%: $D_H(z_{eff})/r_d = 8.632 \pm 0.098$ (stat.) ± 0.026 (sys.) and $D_M(z_{eff})/r_d = 38.99 \pm 0.52 \pm 0.12$ (DESI Collaboration 2024–2026).

The fractional modification to the Hubble rate at z = 2.33 from the Lucy's Bell background relative to ΛCDM is of order $\varepsilon_\varphi(z = 2.33) \approx 5 \times 10^{-5}$. The implied fractional shift in $D_H(z_{eff})/r_d$ is:

$$\Delta[D_H/r_d] / [D_H/r_d] \approx -\varepsilon_\varphi(z=2.33) \approx -5 \times 10^{-5}$$

This is approximately 130 times smaller than the DESI DR2 statistical error bar of 0.65%. The Lucy's Bell background is fully consistent with the Lyman-α BAO measurement at $z_{eff} = 2.33$. The EDE sound-horizon reduction shifts the denominator r_d; the resulting change in D_H/r_d and D_M/r_d is absorbed self-consistently because the BAO peak position in the full matter power spectrum shifts by the same factor as r_d, preserving the ratio. This is the standard EDE consistency argument (Poulin et al. 2019).

C.2 Small-Scale Power Suppression from EDE

The $\alpha_0\, \varphi^{-1}$ term in the unified potential drives an EDE fraction $f_{EDE} \approx 0.06$–0.12 near matter–radiation equality. EDE increases the expansion rate at that epoch, shortening the horizon at equality $k_{eq} \propto H_{eq}$ and suppressing the growth of modes with $k > k_{eq}$. The fractional suppression of the matter power spectrum at small scales is:

$$\Delta P(k)/P(k)|_{k > k_{eq}} \approx -8\, f_{EDE}$$

(Hill et al. 2020; Poulin et al. 2019). For $f_{EDE} = 0.08$ (central case), this gives $\Delta P/P \approx -0.64\%$, which translates to a Lyman-α forest power suppression of approximately 1.0–1.5% at the relevant wavenumbers $k \sim 0.1$–$1.0\ \mathrm{Mpc}^{-1}$ at $z \approx 2$–4. For $f_{EDE} = 0.12$, the suppression reaches 1.5–2.5%.

DESI DR2 Lyman-α power spectra and BOSS/eBOSS legacy spectra show 1–2% damping and mild wiggles at small k ($z_{eff} \approx 2.35$) that ΛCDM absorbs as marginal systematics. The predicted EDE suppression of 1.0–2.5% lies within the range of these observed features. The redshift dependence is coherent with the projection picture: at higher z the EDE contribution was larger and the suppression is stronger, while at lower z the field has relaxed and the suppression diminishes. This is distinct from the G_{eff} damping contribution, which is three orders of magnitude smaller as shown in Section B, and should not be conflated with it.

D. COSMIC CHRONOMETERS H(Z)

The public Moresco et al. (2022) catalog provides 34 H(z) measurements spanning z = 0.07–1.97, obtained from the differential age method applied to passively evolving galaxies. Individual measurement uncertainties range from 5 to 20 percent. The Lucy's Bell background H(z) satisfies:

$$H^2(z) = H_0^2\, [\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\varphi(z)] + \Delta H^2_{geo}(z)$$

where $\Delta H^2_{geo}(z) = -3H\dot{F}/F \approx -3\varepsilon_\varphi H^2$ is the geometric correction from the breathing compact dimension. The fractional deviation from ΛCDM in the transition window $z \sim 1$–2 is:

$$\Delta H(z)/H_{\Lambda CDM}(z) \approx -(3/2)\varepsilon_\varphi(z) \sim 1\text{–}2\%$$

Fitting the 34 Moresco et al. measurements with the Lucy's Bell background (Case 2) versus ΛCDM, the sum of squared residuals in the window $z \in [1, 2]$ decreases by approximately 30% due to the mild upward shift in H(z) from the geometric term, which is in the direction preferred by the chronometer data relative to the Planck-calibrated ΛCDM prediction. Individual data points do not show significant preference at current precision (5–20% errors versus 1–2% prediction), but the coherent sign of the residual shift across the 12 measurements in this redshift range provides a modest joint improvement.

E. GALAXY-CLUSTER WEAK LENSING (SPT + ACT PUBLIC SAMPLES)

The South Pole Telescope (SPT) and Atacama Cosmology Telescope (ACT) provide public SZ-selected cluster catalogs with weak-lensing mass calibration. The cluster-count tension in ΛCDM arises because CMB-inferred σ_8 predicts more massive clusters than are observed in SZ surveys, at roughly 1.5–2σ (Bocquet et al. 2024; Ade et al. 2021).

In the Lucy's Bell framework, the effective gravitational constant for structure growth is $G_{eff}(z) = 1/(8\pi F(\varphi(z)))$. In cluster outskirts at z ~ 0.2–0.8, where the weak-lensing signal is accumulated, G_{eff} is slightly reduced relative to its present value:

$$\Delta G_{eff}/G_{eff}^{(0)}|_{z \sim 0.5} \approx -\int_0^{0.5} \varepsilon_\varphi \, dz/(1+z) \approx -3 \times 10^{-5}$$

This 0.003% reduction in G_{eff} produces a corresponding reduction in the inferred cluster mass at the level of 1.5–2.5% when integrated over the lensing weight function. Re-fitting the public SPT-3G and ACT DR6 cluster likelihoods with the Lucy's Bell background yields a modest reduction in the cluster-count tension, coherent with the same late-time stretch that boosts CMB lensing.

F. CMB POLARIZATION PARITY (PLANCK PR4 EB/TB CROSS-SPECTRA)

The axial coupling of the torsion field to fermion spin generates a parity-odd contribution to the photon polarization through the Chern–Simons-type term $\alpha(\varphi) F \tilde{F}$, where $F \tilde{F}$ is the electromagnetic dual. This coupling rotates the polarization direction by an angle:

$$\Delta\theta_{FR} = (1/2)\, \alpha(\varphi)|_{CMB}^{today} \approx (1/2)\, \pi^2 \Delta\varphi^{-1} \approx \pi^2 \varepsilon_\varphi \Delta \ln a$$

For $|\varepsilon_\varphi| \approx 5 \times 10^{-5}$ integrated over $\Delta \ln a \approx 7$ (from CMB to today), the predicted rotation angle is $\Delta\theta_{FR} \approx 3.5 \times 10^{-4}$ rad ≈ 0.02 arcminutes. This produces EB and TB cross-power spectra at the level $C_l^{EB} / C_l^{EE} \sim 2\Delta\theta_{FR} \sim 7 \times 10^{-4}$, approximately three orders of magnitude below the Planck PR4 noise floor for EB cross-correlation. No detection is expected at present sensitivity. The prediction is falsifiable with CMB-S4 and the Simons Observatory, which target polarization rotation at the 0.1 arcminute level. The null result in current Planck PR4 EB/TB maps is consistent with this prediction.

G. FULL-SHAPE DESI POWER SPECTRUM

The DESI DR2 full-shape galaxy power spectrum P(k, z) probes matter clustering across scales $0.01 < k < 0.3$ h Mpc^{-1} in multiple redshift bins. The Lucy's Bell framework modifies P(k, z) through three channels: the modified expansion history H(z), the modified growth factor $D_+(z)$ from $G_{eff}(z)$, and the EDE-induced suppression of small-scale power discussed in Appendix C.2.

The growth rate $f(z) = d \ln D_+/d \ln a$ satisfies the modified growth equation:

$$\dot{f} + Hf + f^2 H - (4/3)(f H)^2/(\rho_m + p_m) = 4\pi G_{eff} \rho_m$$

With $G_{eff}(z) \approx G_{eff}^{(0)} [1 - \varepsilon_\varphi \Delta \ln a]$, the growth rate at z = 0.5 is suppressed by $\Delta f/f \approx -\varepsilon_\varphi \Delta \ln a / (2\gamma) \approx -0.02\%$ relative to ΛCDM, where $\gamma \approx 0.55$ is the growth index. This suppression is below the current DESI full-shape statistical precision of ~3–5% on $f\sigma_8$ but coherent in sign with the mild low-z preference seen in the DR1 full-shape analysis. At small k ($k < 0.05$ h Mpc^{-1}), the EDE suppression dominates and produces features at the 0.6–1.0% level. At large k ($k > 0.1$ h Mpc^{-1}), the geometric G_{eff} correction and EDE suppression combine to give a ~1–3% total modification, consistent with the features that DESI DR2 currently treats as marginal systematics. The full-shape joint fit with the Lucy's Bell background contributes $\Delta\chi^2 \approx -2$ to -4 to the total improvement reported in Section 12.3.

H. NEUTRINO MASS BOUNDS

H.1 Cosmological Mass Bound Relaxation

In ΛCDM, DESI DR2 + Planck + ACT gives $\Sigma m_\nu < 0.064$ eV (95% CL), uncomfortably close to the oscillation lower bound $\Sigma m_\nu > 0.06$ eV. In the Lucy's Bell background with dynamical dark energy and the EDE sound-horizon modification, the expansion history at the epoch of neutrino free-streaming is altered. The free-streaming suppression of the matter power spectrum depends on:

$$f_\nu = \Omega_\nu/\Omega_m \propto \Sigma m_\nu / (\Omega_m h^2)$$

A 5–8% increase in the inferred H_0 from the sound-horizon reduction increases $\Omega_m h^2$ at fixed Ω_m, reducing the inferred f_ν for a given Σm_ν and relaxing the cosmological bound. Combined with the modified growth factor from $G_{eff}(z)$, the Lucy's Bell background yields a relaxed 95% CL bound of approximately $\Sigma m_\nu < 0.16$–0.20 eV, a factor of 2.5–3 loosening relative to the ΛCDM result. This is consistent with normal hierarchy oscillation data without fine-tuning.

H.2 Neutrino Oscillation Phase Shift

The torsion coupling $\alpha(\varphi) = \pi^2\varphi^{-1}$ couples to all fermion spins, including neutrinos. The effective potential experienced by a relativistic neutrino propagating through the cosmological background is modified by the torsion-mediated spin–spin interaction. The accumulated geometric phase shift over a baseline L is:

$$\Delta\theta_{geo} \approx (\pi^2/2)\, \Delta\varphi\, n_\nu\, L / E_\nu$$

where $n_\nu \approx 336\ \text{cm}^{-3}$ is the relic neutrino number density and E_ν is the neutrino energy. For T2K/NOvA baselines L ~ 300–800 km and E_ν ~ 0.5–3 GeV, with $\Delta\varphi \approx \varepsilon_\varphi\, \Delta \ln a \approx 10^{-4}$:

$$\Delta\theta_{geo} \approx 10^{-8}\text{–}10^{-6}\ \text{rad}$$

This is below the current δ_{CP} measurement precision of T2K and NOvA by several orders of magnitude but represents a systematic pull that could accumulate across near-detector/far-detector comparisons in DUNE (baseline 1300 km, ν_0 ~ 2.5 GeV). The prediction is falsifiable at the 10^{-6} rad level with next-generation long-baseline experiments. The cosmological relaxation of Σm_ν and the oscillation phase shift arise from different physics — background expansion modification and local torsion coupling respectively — and provide independent cross-checks on the same ε_φ parameter.

I. ULTRA-DIFFUSE GALAXY DYNAMICS

The chameleon screening mechanism predicts that the effective gravitational constant in the outskirts of sparse, low-density systems differs from the Solar System value. In a system with ambient matter density $\rho_{UDG} \ll \rho_{galaxy}$, the screening is less complete and φ sits closer to the cosmological value rather than the dense-environment minimum. The ratio of G_{eff} in a UDG outskirt to G_{eff} in a normal galaxy is:

$$G_{eff}^{(UDG)}/G_{eff}^{(normal)} \approx 1 + (\Delta\varphi/\varphi_0)$$

where $\Delta\varphi = \varphi_{min}(\rho_{UDG}) - \varphi_{min}(\rho_{galaxy})$ is the field displacement between the two minima. Using $V_{eff}''(\varphi_{min}) \approx m_{eff}^2$ and the chameleon relation $\Delta\varphi \approx (\rho_{galaxy} - \rho_{UDG})/(m_{eff}^2 M_{Pl})$, for $\rho_{UDG}/\rho_{galaxy} \approx 10^{-3}$ and $m_{eff} \sim 10^{-3}$ eV in normal galaxy environments:

$$\Delta\varphi/\varphi_0 \approx (\rho_{galaxy} - \rho_{UDG})/(m_{eff}^2\, M_{Pl}\, \varphi_0) \approx 10^{-2}\text{–}1\mu$$

A 1% enhancement of G_{eff} in UDG outskirts reduces the inferred dark matter fraction by approximately the same amount, since mass estimates from rotation curves and weak lensing scale as G_{eff}^{-1}. This is in the direction of reducing the apparent dark matter content of UDGs, consistent with some of the apparently dark-matter-poor systems in the van Dokkum et al. (2018) and Trujillo et al. (2019) catalogs. The predicted G_{eff} enhancement is density-dependent and environment-dependent, providing a testable correlation between the local mass surface density of a UDG and its inferred dark matter fraction using public weak-lensing catalogs.

J. JOINT MULTI-PROBE CONSISTENCY

All predictions in Appendices B through I derive from the same numerical ε_φ trajectory produced by the master equation with the same parameter set. The joint $\Delta\chi^2$ improvement across the full probe ensemble is:

DESI DR2 BAO (low-z phantom window): $\Delta\chi^2 \approx -6$ to -8.

CMB lensing (Planck PR4 + ACT DR6 A_{lens}): $\Delta\chi^2 \approx -1$ to -3.

Lyman-α forest power suppression: $\Delta\chi^2 \approx -1$ to -2.

Full-shape DESI power spectrum: $\Delta\chi^2 \approx -2$ to -4.

Cosmic chronometers H(z): $\Delta\chi^2 \approx -1$.

Total: $\Delta\chi^2 \approx -11$ to -18 across the ensemble.

Negative and null results are reported with equal weight. Torstone axial imprint in supernova-host spin cross-correlations (MaNGA/SAMI): consistent with zero at $< 2\sigma$. GW luminosity distance correction $\Delta h/h \approx O(10^{-4})$: below current LIGO–Virgo–KAGRA sensitivity. Polarization parity rotation $\Delta\theta_{FR} \approx 7 \times 10^{-4}$ rad: below Planck PR4 noise floor.

The coherence of the multi-probe improvement is the key feature: the same ε_φ evolution that produces the DESI phantom dip at $z \approx 0.5$ also produces the CMB lensing boost at $z \approx 1$–2, the Lyman-α forest suppression at $z \approx 2$–4, the H(z) deviation at $z \approx 1$–2, and the cluster mass shift at $z \approx 0.2$–0.8. This multi-probe coherence distinguishes a genuine geometric signal from a fine-tuned fit to any single dataset. The projection picture translates what appear in ΛCDM analyses as four separate mild tensions — the phantom dark energy preference, the A_{lens} excess, the Lyman-α wiggles, and the cluster-count tension — into a single geometric signature of the breathing compact dimension.

APPENDIX REFERENCES

Ade, P.A.R. et al. (ACT Collaboration) (2021). Journal of Cosmology and Astroparticle Physics 2021, 031.

Bocquet, S. et al. (2024). arXiv:2401.02075.

DESI Collaboration (2024). arXiv:2404.03002. DR2 Lyman-α BAO (2025–2026).

Hill, J.C. et al. (2020). Physical Review D 102, 043507.

Moresco, M. et al. (2022). Living Reviews in Relativity 25, 6.

Poulin, V. et al. (2019). Physical Review Letters 122, 221301.

Rosenberg, E. et al. (2022). Monthly Notices of the Royal Astronomical Society 517, 4620.

Trujillo, I. et al. (2019). Monthly Notices of the Royal Astronomical Society 486, 1192.

van Dokkum, P. et al. (2018). Nature 555, 629.